2.6/26)

$$\lim_{h\to 0}\frac{\sin h}{(1-\cos h)}\frac{(1+\cos h)}{(1+\cos h)}$$

$$=\lim_{h\to 0}\frac{(\sin h)(1+\cos h)}{1-\cos^2 h}$$

$$=\lim_{h\to 0}\frac{(\sin h)(1+\cos h)}{\sin^2 h}$$

$$\text{"}\tfrac{2}{0}\text{"}=\underbrace{\lim_{h\to 0}(1)}_{}\frac{(1+\cos h)}{\sin h}$$

L DNE L $+\infty$ L $-\infty$

$$\lim_{h\to 0^-}\frac{1+\cos h^{\to 2}}{\sin h}=-\infty$$

$$\lim_{h\to 0^+}\frac{1+\cos h}{\sin h}=+\infty$$

$\therefore$ original limit DNE

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to 0}\frac{x}{\sin x}=1$$

$$\lim_{x\to 0}\frac{x}{1-\cos x}=$$

DNE
$+\infty$
$-\infty$

$$\sin^2+\cos^2=1$$

$$\sin^2=1-\cos^2$$

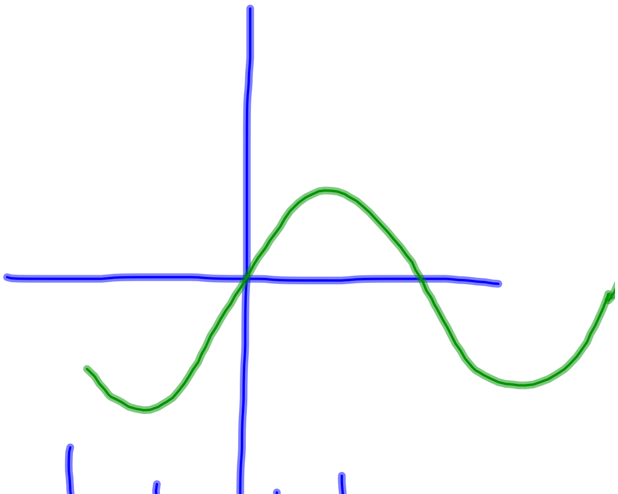
27) $\lim\limits_{\theta \to 0} \dfrac{\theta^2}{1-\cos\theta}$

$$\lim_{\theta \to 0} \left(\frac{\theta^2}{\sin^2\theta}\right)\left(\frac{1+\cos\theta}{1}\right)$$

$$= \left(\lim_{\theta \to 0} \frac{\theta}{\sin\theta}\right) \cdot \left(\lim_{\theta \to 0} \frac{\theta}{\sin\theta}\right) \cdot \lim_{\theta \to 0}\left(\frac{1+\cos\theta}{1}\right)$$

28) $\lim_{x \to 0} \frac{x}{\cos\left(\frac{\pi}{2}-x\right)}$

$\cos\left(-\left(x-\frac{\pi}{2}\right)\right)$



$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\cos\left(\frac{\pi}{2}-x\right) = \underbrace{\cos\frac{\pi}{2}\cos x}_{0} + \sin\frac{\pi}{2}\sin x$

$+\sin x$

$\cos(2x)$

$\cos(x+x)$

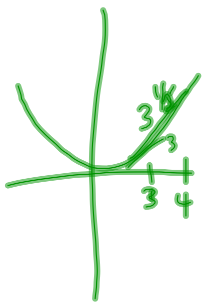
3.1/7) $f(x)=\frac{1}{2}x^2$; $x_0=3$; $x_1=4$

a) avg rate of chg: $\dfrac{f(4)-f(3)}{4-3}=\dfrac{8-\frac{9}{2}}{1}=\dfrac{7}{2}$

b) inst rate of chg at $x_0=3$ $=\lim\limits_{x_1\to 3}\dfrac{f(x_1)-f(3)}{x_1-3}$

$$=\lim_{x_1\to 3}\frac{\frac{1}{2}(x_1)^2-\frac{1}{2}(3)^2}{x_1-3}$$

$$=\lim_{x_1\to 3}\frac{\frac{1}{2}x_1^2-\frac{9}{2}}{x_1-3}=\lim_{x_1\to 3}\frac{\frac{1}{2}(x_1^2-9)}{x_1-3}$$



$$=\lim_{x_1\to 3}\frac{\frac{1}{2}(x_1-3)(x_1+3)}{x_1-3}=\lim_{x_1\to 3}\frac{1}{2}(x_1+3)$$

$$=\frac{1}{2}(3+3)=3$$

c) inst rate of chg at x_0

p173

$$\lim_{x_1\to x_0}\frac{f(x_1)-f(x_0)}{x_1-x_0}=$$

$$\lim_{x_1\to x_0}\frac{\frac{1}{2}x_1^2-\frac{1}{2}x_0^2}{x_1-x_0}=\lim_{x_1\to x_0}\frac{\frac{1}{2}(x_1^2-x_0^2)}{x_1-x_0}$$

$$=\lim_{x_1\to x_0}\frac{\frac{1}{2}(x_1-x_0)(x_1+x_0)}{x_1-x_0}=\lim_{x_1\to x_0}\frac{1}{2}(x_1+x_0)$$

$$=\frac{1}{2}(x_0+x_0)=\frac{1}{2}(2x_0)=x_0$$

8) x^3; 1; 2;

$$(a^3-b^3)=(a-b)(a^2+ab+b^2)$$

$$(a^3+b^3)=(a+b)(a^2-ab+b^2)$$

a) $\underset{\text{r.o.c.}}{\text{avg}} = \frac{f(2)-f(1)}{2-1} = \frac{2^3-1^3}{1} = \frac{8-1}{1}$

$= 7$

---

b) $\underset{\text{roc@1}}{\text{inst}} = \lim_{x_1 \to 1} \frac{f(x_1)-f(1)}{x_1-1}$

$= \lim_{x_1\to 1} \frac{x_1^3-1}{x_1-1} = \lim_{x_1\to 1} \frac{(x_1-1)(x_1^2+x_1+1)}{x_1-1}$

$= \lim_{x_1\to 1} x_1^2+x_1+1 = 1+1+1 = 3$

---

c) x^3; 1; 2.

$\underset{\text{roc}}{\text{inst}} = \lim_{x_1\to x_0} \frac{x_1^3-x_0^3}{x_1-x_0} = \lim_{x_1\to x_0} \frac{(x_1-x_0)(x_1^2+x_1x_0+x_0^2)}{x_1-x_0}$

$= \lim_{x_1\to x_0} x_1^2+x_1x_0+x_0^2 = x_0^2+x_0^2+x_0^2$

$= 3x_0^2$

#9 idea

$$\frac{\frac{1}{x}-\frac{1}{y}}{x-y} = \frac{\frac{y}{xy}-\frac{x}{xy}}{x-y}$$

$$= \frac{\frac{y-x}{xy}\left(\frac{1}{x-y}\right)}{x-y\left(\frac{1}{x-y}\right)}$$

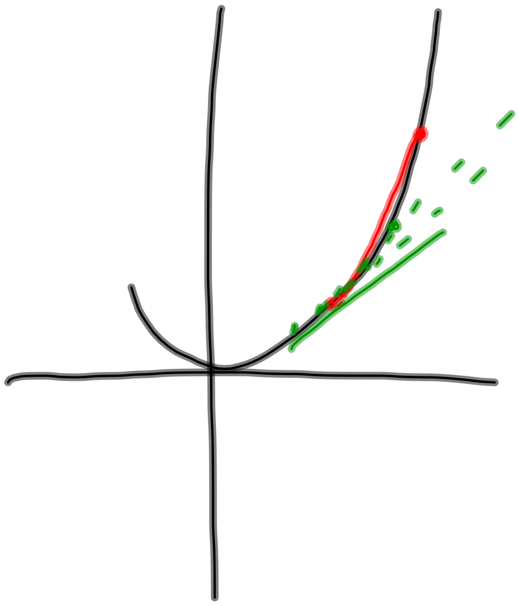
$$= \frac{(y-x)}{xy(x-y)} = \frac{-1}{xy}$$

3.2

Idea #1: inst r.o.c. at $x = x_0$ $\equiv$ the derivative of f at $x = x_0$

$$\equiv f'(x_0)$$

Idea 2: to write an equation of a line...



3.1/7) $f(x)=\frac{1}{2}x^2$; $x_0=3$; $x_1=4$

a) avg. r.o.c. fr. x=3 to x=4 : $\frac{f(x_1)-f(x_0)}{x_1-x_0}=\frac{f(4)-f(3)}{4-3}$

$\frac{1}{2}3^2$

$=\frac{8-\frac{9}{2}}{1}=\frac{7}{2}$

b) inst roc @ x=3 $=\lim_{x_1\to x_0}\frac{f(x_1)-f(x_0)}{x_1-x_0}=\lim_{x_1\to 3}\frac{f(x_1)-f(3)}{x_1-3}$

p13

$=\lim_{x_1\to 3}\frac{\frac{1}{2}x_1^2-\frac{9}{2}}{x_1-3}=\lim_{x_1\to 3}\frac{\frac{1}{2}(x_1^2-9)}{x_1-3}$

$=\lim_{x_1\to 3}\frac{\frac{1}{2}(x_1-3)(x_1+3)}{x_1-3}$

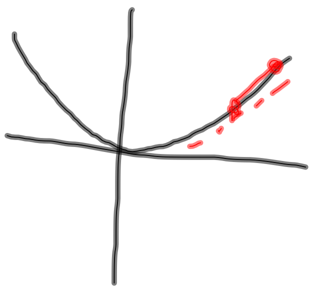
$=\lim_{x_1\to 3}\frac{1}{2}(x_1+3)=\frac{1}{2}(3+3)=3$

c) inst r.o.c at x_0 $\lim_{x_1\to x_0}\frac{f(x_1)-f(x_0)}{x_1-x_0}$

$=\lim_{x_1\to x_0}\frac{\frac{1}{2}x_1^2-\frac{1}{2}x_0^2}{x_1-x_0}=\lim_{x_1\to x_0}\frac{\frac{1}{2}(x_1^2-x_0^2)}{x_1-x_0}$

$=\lim_{x_1\to x_0}\frac{\frac{1}{2}(x_1-x_0)(x_1+x_0)}{x_1-x_0}=\lim_{x_1\to x_0}\frac{1}{2}(x_1+x_0)$

$=\frac{1}{2}(x_0+x_0)=\frac{1}{2}(2x_0)=x_0$



8) $f(x)=x^3$; $x_0=1$; $x_1=2$

$$(a^3-b^3)=(a-b)(a^2+ab+b^2)$$
$$(a^3+b^3)=(a+b)(a^2-ab+b^2)$$

a) avg roc bet 1,2 $= \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{f(2)-f(1)}{2-1}$

$= \dfrac{8-1}{1} = 7$

b) inst roc @ $x_0=1$ $= \lim\limits_{x_1\to 1} \dfrac{f(x_1)-f(1)}{x_1-1} = \lim\limits_{x_1\to 1} \dfrac{x_1^3-1^3}{x_1-1}$

$= \lim\limits_{x_1\to 1} \dfrac{(x_1-1)(x_1^2+x_1+1)}{x_1-1} = \lim\limits_{x_1\to 1} x_1^2+x_1+1 = 1+1+1=3$

c) inst roc @ x_0 $= \lim\limits_{x_1\to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1\to x_0} \dfrac{x_1^3-x_0^3}{x_1-x_0} =$

$\lim\limits_{x_1\to x_0} \dfrac{(x_1-x_0)(x_1^2+x_1x_0+x_0^2)}{x_1-x_0}$

$= \lim\limits_{x_1\to x_0} x_1^2+x_1x_0+x_0^2 = x_0^2+x_0^2+x_0^2 = 3x_0^2$

9) $f(x)=\frac{1}{x}$; 2, 3

$$\frac{\frac{1}{a}-\frac{1}{b}}{a-b}=\frac{\frac{b}{ab}-\frac{a}{ab}}{a-b}=\frac{\frac{b-a}{ab}\left(\frac{1}{a-b}\right)}{a-b\left(\frac{1}{a-b}\right)}$$

$$=\frac{b-a}{ab(a-b)}=\frac{-1\,(a-b)}{ab\,(a-b)}$$

3.2)

Idea:

If I know a rule that tells me what the inst. r.o.c. of a f^n wrt x is at ANY x-value...

that is a FUNCTION.

Now I have:

$f(x)$

$f^{-1}(x)$ (potentially)

instroc function... $f'(x)$

this f^n is called the DERIVATIVE.

Idea:

the instaneous roc of a f^n wrt x
at a pt c
IS def^n to be the <u>slope</u>
of the tangent line
to the f^n $f(x)$ at $x=c$.

AND, now, with the slope
AND a pt $(c, f(c))$
I can write an
equation of a tangent line.

Extra Extra

Many of the surprise quiz "problems" are related to one idea:

$$(a+b)^n \neq a^n + b^n$$

(unless $n=1$)

for example ($n=2$)

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2 \neq a^2 + b^2$$

And ($n=\frac{1}{2}$)

$$\underset{\sqrt{a+b}}{\underset{\parallel}{(a+b)^{1/2}}} \neq a^{1/2} + b^{1/2}$$

[if it did, Pythagorean Theorem would be simpler]